

Magnetic black holes with higher-order curvature and gauge corrections in even dimensions

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ABSTRACT: We obtain magnetic black-hole solutions in arbitrary $n(\geq 4)$ even dimensions for an action given by the Einstein-Gauss-Bonnet-Maxwell- Λ pieces with the F^4 gauge-correction terms. This action arises in the low energy limit of heterotic string theory with constant dilaton and vanishing higher form fields. The spacetime is assumed to be a warped product $\mathcal{M}^2 \times \mathcal{K}^{n-2}$, where \mathcal{K}^{n-2} is a $(n-2)$ -dimensional Einstein space satisfying a condition on its Weyl tensor, originally considered by Dotti and Gleiser. Under a few reasonable assumptions, we establish the generalized Jebsen-Birkhoff theorem for the magnetic solution in the case where the orbit of the warp factor on \mathcal{K}^{n-2} is non-null. We prove that such magnetic solutions do not exist in odd dimensions. In contrast, in even dimensions, we obtain an explicit solution in the case where \mathcal{K}^{n-2} is a product manifold of $(n-2)/2$ two-dimensional maximally symmetric spaces with the same constant warp factors. In this latter case, we show that the global structure of the spacetime sharply depends on the existence of the gauge-correction terms as well as the number of spacetime dimensions.

Contents

1. Introduction	2
2. Einstein-Gauss-Bonnet-Maxwell-Λ system with gauge-correction terms	3
2.1 Preliminaries	4
2.2 Ansätze	5
3. Magnetic solutions	6
3.1 The Jebsen-Birkhoff theorem	6
3.2 Non-existence in odd dimensions	9
3.3 Exact magnetic solutions in even dimensions	9
4. Properties of the magnetic black holes with gauge corrections	11
4.1 Curvature singularities	11
4.2 Asymptotic structure	12
4.3 Energy conditions	13
4.4 Black hole configurations	13
4.4.1 General relativity without gauge corrections	15
4.4.2 General relativity with gauge corrections	15
4.4.3 Einstein-Gauss-Bonnet gravity without gauge corrections	16
4.4.4 Einstein-Gauss-Bonnet gravity with gauge corrections	17
5. Summary and discussions	18

1. Introduction

What is the description of a black hole in the quantum theory of gravity? The answer to this question is one of the ultimate goals of modern physics. String theory which is consistently formulated in higher dimensions is a promising candidate of a unified theory. In string theory, the extra dimensions are usually considered to be compactified, and as a result, the effect of string theory is negligible for large astrophysical black holes. However, in order to discuss the formation of tiny black holes in the upcoming Large Hadron Collider (LHC) or the final fate of an evaporating black hole by the Hawking radiation, the effect of string theory cannot be neglected. If the horizon size becomes comparable to the curvature radius of the compactified extra dimensions, the black hole effectively becomes higher-dimensional. Although the non-perturbative aspects of string theory have been intensively investigated in recent years focusing on the conjecture of anti-de Sitter(AdS)/conformal field theory(CFT) correspondence [1], the full description of a black hole in string theory is still far from complete.

Another possible approach to study the string effect is to study black holes in the low-energy classical theory. Higher-dimensional general relativity is realized as the lowest order in the Regge slope expansion of strings. Then, it is known that the higher-curvature terms appear as the next stringy compensation. Among five types of string theories, there appears the so-called Gauss-Bonnet term in the heterotic string case [2, 3], which is a ghost-free and renormalizable combination of the quadratic curvature terms [4]. The active study of the Gauss-Bonnet black holes has its roots in the discovery of the spherically symmetric vacuum Boulware-Deser-Wheeler solution [5], which is the counterpart of the Tangherlini solution in general relativity [6]. However, in spite of the considerable progress in recent years on this subject, even the stationary axisymmetric rotating vacuum black-hole solution, namely the counterpart of the Myers-Perry solution in general relativity [7], has not been obtained yet. (See [8] for recent reviews.)

Since gauge fields are fundamental in the standard model, black holes with gauge fields are also important from the viewpoint of string theory. The Gauss-Bonnet black-hole solution with Maxwell electric charge was obtained by Wiltshire [9] and has been generalized to the topological case with a cosmological constant [10, 11, 12]. Indeed, in the low-energy limit of heterotic string theory, the higher-order correction terms appear also for the Maxwell gauge field [3]. Therefore, in order to study the semi-classical aspects of black holes, it is fair to consider not only the correction terms arising from the gravity side but also those related to the gauge field. This is one of the motivations of the present paper. Here we will be concerned with the Einstein equations supplemented by the Gauss-Bonnet term with a source provided by the Maxwell field with the F^4 gauge-correction terms. The motivation of considering such Lagrangian are multiple. Firstly, the electrically charged Gauss-Bonnet black holes with the higher F -terms is a current well-studied topic [13, 14, 15]. In Ref. [14], the effects of the F^4 gauge-correction terms on the thermodynamical aspects of black holes have been fully investigated. In addition, the Lagrangian adopted in [14] is considered as an interesting model in the low-energy limit of heterotic string theory since it contains the Lagrangian in the low-energy limit of the ten-dimensional $E_8 \times E_8$ or $SO(32)$ heterotic string theory. To be more complete, the action considered in [14] also arises in four dimensions from the corrections to the magnetically charged string black holes [16] by setting the dilaton to be constant.

In this paper, we are interested in magnetic black holes in arbitrary $n(\geq 4)$ dimensions. As is well-known, the Maxwell electromagnetic field is a fundamental gauge field in physics which presents some attractive features in four dimensions. Among other things, the electro-magnetic 2-form duality as well as the conformal invariance of the Maxwell action are effective only in four dimensions. In higher dimensions, maybe because of the lack of these properties, the Maxwell field is not well tamed. As an appealing example to illustrate this fact, the higher-dimensional version of the Kerr-Newman solution has not been obtained in general relativity so far. (See [17] for discussions.) Another example is provided with the study of magnetic black holes. Indeed, even if some results are known [18], the problem of finding such solutions even in the case of spacetimes locally $\mathcal{M}^2 \times \mathcal{K}^{n-2}$ is still an open problem. (Here \mathcal{K}^{n-2} is a $(n-2)$ -dimensional Einstein space.) In this case, the difficulties may arise because the number of the magnetic components of the Faraday tensor (in contrast with the standard electric solution) grows with the spacetime dimensions.

In general relativity, it is well-known that replacing the $(n-2)$ -dimensional space of positive constant curvature in the Schwarzschild-Tangherlini spacetime by *any* $(n-2)$ -dimensional Einstein space with positive curvature will still provide a solution of the vacuum Einstein equations. However, this is not the case in the presence of the Gauss-Bonnet term. The reason is that, unlike the Einstein tensor, the quadratic Gauss-Bonnet tensor contains the Riemann tensors explicitly, and hence it gives a more severe constraint on the $(n-2)$ -dimensional Einstein space. In the Boulware-Deser-Wheeler vacuum spacetime, \mathcal{K}^{n-2} is maximally symmetric, namely a $(n-2)$ -dimensional space of positive constant curvature. Considering a $(n-2)$ -dimensional Einstein space for \mathcal{K}^{n-2} in Einstein-Gauss-Bonnet gravity, Dotti and Gleiser derived a consistency condition on the Weyl tensor on \mathcal{K}^{n-2} with the field equations and obtained an exact vacuum black-hole solution [19]. The effect of the Weyl tensor appears in the metric function and makes the spacetime geometry quite non-trivial. In this paper, we will consider the Dotti-Gleiser condition as an assumption in order to obtain magnetic black-hole solutions in the case where \mathcal{K}^{n-2} is an Einstein space. (Both static and dynamical aspects of Gauss-Bonnet black holes with this class of non-constant curvature horizons have been recently studied in [20].)

The plan of the paper is organized as follows. In the next section, we present the model and clearly state our assumptions. In section III, we obtain the unique possible form of the metric compatible with the magnetic field and we will prove the non-existence of the magnetic solutions in odd dimensions. We also derive explicit solutions in even dimensions. In the section IV, we discuss the properties of the solution and show that the black-hole configurations arise for a particular range of the parameters. In the section V, we summarize our results. Our basic notations follow [21]. The conventions of curvature tensors are $[\nabla_\rho, \nabla_\sigma]V^\mu = R^\mu{}_{\nu\rho\sigma}V^\nu$ and $R_{\mu\nu} = R^\rho{}_{\mu\rho\nu}$, where ∇_ν is the covariant derivative. The Minkowski metric is taken to be the mostly plus sign, and Roman indices run over all spacetime indices. We adopt the units in which only the n -dimensional gravitational constant G_n is retained.

2. Einstein-Gauss-Bonnet-Maxwell- Λ system with gauge-correction terms

In this section we consider the Einstein action supplemented by the cosmological constant and the Gauss-Bonnet term in arbitrary dimensions. The matter source is provided by the Maxwell action and

the F^4 gauge-corrections terms built up with the Faraday tensor. After deriving the field equations and explaining the origin of such an action, we will assume that the spacetime geometry is given by a warped product $\mathcal{M}^2 \times \mathcal{K}^{n-2}$, where \mathcal{K}^{n-2} is a $(n-2)$ -dimensional Einstein space satisfying a certain condition presented below.

2.1 Preliminaries

In arbitrary dimensions $n(\geq 4)$, we consider the following action

$$S = S_{\text{gravity}} + S_{\text{matter}}, \quad (2.1)$$

$$S_{\text{gravity}} = \int d^n x \sqrt{-\det(g_{\mu\nu})} \left[\frac{1}{2\kappa_n^2} (R - 2\Lambda + \alpha L_{GB}) \right], \quad (2.2)$$

$$S_{\text{matter}} = -\frac{1}{4g^2} \int d^n x \sqrt{-\det(g_{\mu\nu})} F_{\mu\nu} F^{\mu\nu} + \int d^n x \sqrt{-\det(g_{\mu\nu})} \left[c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} \right], \quad (2.3)$$

where $L_{GB} := R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the Gauss-Bonnet Lagrangian and $\kappa_n := \sqrt{8\pi G_n}$, where G_n is n -dimensional gravitational constant. The Maxwell field strength, or the Faraday tensor, is given by $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$ where A^μ is the vector potential. The parameters α , g , c_1 , and c_2 are real constants.

The action (2.1) with $n = 10$ arises in the low-energy limit of heterotic string theory with constant dilaton. Indeed, in the low-energy limit of the ten-dimensional $E_8 \times E_8$ or $SO(32)$ heterotic string theory with a constant dilaton ϕ_0 and turning off the higher form fields, the following Lagrangian is realized [3, 13]:

$$L_{\text{low}} = \frac{1}{2\kappa_{10}^2} R - \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha' h}{16\kappa_{10}^2} L_{GB} - \frac{3\alpha' h \kappa_{10}^2}{64} \left[(F_{\mu\nu} F^{\mu\nu})^2 - 4F_{\mu\nu} F^{\nu\rho} F_{\rho\sigma} F^{\sigma\mu} \right], \quad (2.4)$$

$$h := e^{-\kappa_{10}\phi_0/\sqrt{2}}, \quad (2.5)$$

where the constant α' stands for the inverse string tension. The above Lagrangian is a particular case of the one considered here (2.1) with $n = 10$, $\Lambda = 0$, $\alpha = \alpha' h/8$, $c_1 = -3\alpha' h \kappa_{10}^2/64 (< 0)$, and $c_2 = -4c_1$.

The gravitational equations following from the variation of the action (2.1) read

$$\mathcal{G}^\mu{}_\nu := G^\mu{}_\nu + \alpha H^\mu{}_\nu + \Lambda \delta^\mu{}_\nu = \kappa_n^2 T^\mu{}_\nu, \quad (2.6)$$

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad (2.7)$$

$$H_{\mu\nu} := 2 \left(R R_{\mu\nu} - 2R_{\mu\alpha} R^\alpha{}_\nu - 2R^{\alpha\beta} R_{\mu\alpha\nu\beta} + R_\mu{}^{\alpha\beta\gamma} R_{\nu\alpha\beta\gamma} \right) - \frac{1}{2} g_{\mu\nu} L_{GB}, \quad (2.8)$$

where the energy-momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = \frac{1}{g^2} \left(F_{\mu\rho} F_{\nu}{}^{\rho} - \frac{1}{4} g_{\mu\nu} \mathcal{F} \right) + 2c_1 \left(\frac{1}{2} g_{\mu\nu} \mathcal{F}^2 - 4 F_{\mu\rho} F_{\nu}{}^{\rho} \mathcal{F} \right) + 2c_2 \left(\frac{1}{2} g_{\mu\nu} F_{\lambda\rho} F^{\rho\sigma} F_{\sigma\tau} F^{\tau\lambda} - 4 F_{\mu\rho} F^{\rho\sigma} F_{\sigma\tau} F^{\tau}{}_{\nu} \right), \quad (2.9)$$

$$\mathcal{F} := F_{\mu\nu} F^{\mu\nu}. \quad (2.10)$$

The Maxwell equation with the gauge-correction terms reads

$$\nabla_{\nu} \left(-\frac{1}{g^2} F^{\mu\nu} + 8c_1 \mathcal{F} F^{\mu\nu} + 8c_2 F^{\mu\rho} F_{\rho\sigma} F^{\sigma\nu} \right) = 0. \quad (2.11)$$

2.2 Ansätze

Now we consider an Ansatz for the spacetime geometry such that the n -dimensional spacetime $(\mathcal{M}^n, g_{\mu\nu})$ is given by a warped product of an $(n-2)$ -dimensional Einstein space (K^{n-2}, γ_{ij}) and a two-dimensional orbit spacetime (M^2, g_{ab}) under the isometries of (K^{n-2}, γ_{ij}) . Namely, the line element is given by

$$g_{\mu\nu} dx^{\mu} dx^{\nu} = g_{ab}(y) dy^a dy^b + r^2(y) \gamma_{ij}(z) dz^i dz^j, \quad (2.12)$$

where $a, b = 0, 1$ while $i, j = 2, \dots, n-1$. Here r is a scalar on (M^2, g_{ab}) and γ_{ij} is the metric on (K^{n-2}, γ_{ij}) with its sectional curvature $k = \pm 1, 0$.

The $(n-2)$ -dimensional Einstein space satisfies

$$\overset{(n-2)}{R}_{ijkl} = \overset{(n-2)}{C}_{ijkl} + k(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}), \quad (2.13)$$

where $\overset{(n-2)}{C}_{ijkl}$ is the Weyl tensor. The superscript $(n-2)$ means that the geometrical quantity are defined on (K^{n-2}, γ_{ij}) . Note that if the Weyl tensor is identically zero, (K^{n-2}, γ_{ij}) is a space of constant curvature. The Riemann tensor is contracted to give

$$\overset{(n-2)}{R}_{ij} = k(n-3)\gamma_{ij}, \quad \overset{(n-2)}{R} = k(n-2)(n-3). \quad (2.14)$$

In this paper, we consider an Einstein space (K^{n-2}, γ_{ij}) satisfying the following condition

$$\overset{(n-2)}{C}{}^{iklm} \overset{(n-2)}{C}{}_{jklm} = \Theta \delta^i_j, \quad (2.15)$$

where Θ is constant and non-negative since (K^{n-2}, γ_{ij}) is an Euclidean space. The condition (2.15) was originally introduced by Dotti and Gleiser for the compatibility with the Einstein-Gauss-Bonnet equations (2.6) with generic coupling constants and called the *horizon condition* [19]. (See [22] for the classification of the submanifold depending on the coupling constants.) Decomposed geometric tensors of this class of spacetime are presented in Appendix B in [20].

Nontrivial examples of the Einstein space satisfying the horizon condition (2.15) are presented in [20, 23, 24]. An example of the Einstein space satisfying the horizon condition that we will consider below is given by a product space of arbitrary number of two-dimensional spaces of constant curvature K^2 with the same warp factor. In this case, the constant Θ is given by $\Theta = 2(n-3)(n-4)k^2$. (This is a particular case of the result shown in Appendix A in [20].) It is easy to see that if K^2 is flat, the resulting space (K^{n-2}, γ_{ij}) is nothing but a $(n-2)$ -dimensional flat space. Although the authors do not know concrete non-trivial examples of the Einstein space with non-zero Θ for some sets of k and n (for $k=0$ with any n , for example), we also consider such cases in this paper.

Since \mathcal{G}_{ij} is proportional to γ_{ij} , the energy-momentum tensor must have the following form

$$T_{\mu\nu}dx^\mu dx^\nu = T_{ab}(y)dy^a dy^b + p(y)r^2(y)\gamma_{ij}dz^i dz^j, \quad (2.16)$$

where $p(y)$ is a scalar function on (M^2, g_{ab}) . In analogy with the spacetime Ansatz (2.12), we look for an electromagnetic field of the form

$$A_\mu dx^\mu = A_a(y)dy^a + A_i(z)dz^i, \quad (2.17)$$

which in turn implies that the Faraday tensor reads

$$F_{\mu\nu}dx^\mu \wedge dx^\nu = F_{ab}(y)dy^a \wedge dy^b + F_{ij}(z)dz^i \wedge dz^j. \quad (2.18)$$

Here $F_{ab}(y)$ and $F_{ij}(z)$ are identified with the electric and magnetic components, respectively. For the magnetic component, we add the following assumption

$$\gamma^{kl}F_{ik}F_{jl} = C^2\gamma_{ij}, \quad (2.19)$$

where C is a constant. The origin of this condition lies in the fact that in the case without the gauge-correction terms this condition is not an input but rather a consequence of the field equations. Hence, it is reasonable to assume Eq. (2.19) in the presence of the gauge corrections.

3. Magnetic solutions

In this section, we obtain magnetic solutions under the assumptions presented in the previous section. We first determine the possible form of the metric in the next subsection, namely we establish a generalized version of the Jebsen-Birkhoff theorem. This can be performed without showing the existence of the non-trivial solution of the gauge-corrected Maxwell equation. The problem of the existence will be studied subsequently.

3.1 The Jebsen-Birkhoff theorem

In the vacuum case with $\Theta = 0$, the generalized Jebsen-Birkhoff theorem was shown under the assumption that $(D_a r)(D^a r) \neq 0$ in [9, 25, 26], where D_a is the covariant derivative on (M^2, g_{ab}) . For the null case $(D_a r)(D^a r) = 0$ [27], on the other hand, there are the Nariai-Bertotti-Robinson type solutions [28] as in the case with or without the Maxwell field in general relativity [29] and in

the Einstein-Gauss-Bonnet gravity [24, 30, 31]. In the case of $\Theta \neq 0$, the generalized Jebsen-Birkhoff theorem for the vacuum case was shown in [20].

Here, we only consider the case where $(D_a r)(D^a r) \neq 0$, which in turn implies that the line element of the spacetime may be written as

$$ds^2 = -g(t, r)e^{-\delta(t, r)}dt^2 + \frac{1}{g(t, r)}dr^2 + r^2\gamma_{ij}dz^i dz^j. \quad (3.1)$$

In this coordinate system, the Maxwell invariant scalar reads

$$\mathcal{F} = 2F_{tr}F^{tr} + \frac{(n-2)C^2}{r^4}, \quad (3.2)$$

while the components of the energy-momentum tensor are given by

$$T^a_b = \left[\frac{1}{2g^2} \left(F_{tr}F^{tr} - \frac{(n-2)C^2}{2r^4} \right) + 2c_1 \left(-3F_{tr}F^{tr} + \frac{(n-2)C^2}{2r^4} \right) \left(2F_{tr}F^{tr} + \frac{(n-2)C^2}{r^4} \right) + 2c_2 \left(-3(F_{tr}F^{tr})^2 + \frac{(n-2)C^4}{2r^8} \right) \right] \delta^a_b, \quad (3.3)$$

$$T^i_j = \left[-\frac{1}{2g^2} \left(F_{tr}F^{tr} + \frac{(n-6)C^2}{2r^4} \right) + 2c_1 \left(F_{tr}F^{tr} + \frac{(n-10)C^2}{2r^4} \right) \left(2F_{tr}F^{tr} + \frac{(n-2)C^2}{r^4} \right) + 2c_2 \left((F_{tr}F^{tr})^2 + \frac{(n-10)C^4}{2r^8} \right) \right] \delta^i_j. \quad (3.4)$$

In the above expressions, we put both the electric component F_{ab} and the magnetic component F_{ij} . The purely electric case, i.e., $F_{ij} \equiv 0$, was fully studied in [14].

Hereafter we consider the purely magnetic case, i.e., $F_{ab} \equiv 0$. Then, the energy-momentum tensor becomes

$$T^a_b = \left[-\frac{(n-2)C^2}{4g^2r^4} + \frac{(n-2)\{(n-2)c_1 + c_2\}C^4}{r^8} \right] \delta^a_b, \quad (3.5)$$

$$T^i_j = \left[-\frac{(n-6)C^2}{4g^2r^4} + \frac{(n-10)\{(n-2)c_1 + c_2\}C^4}{r^8} \right] \delta^i_j. \quad (3.6)$$

The fact that $T^a_b \propto \delta^a_b$ implies $\mathcal{G}^t_r = \mathcal{G}^r_t = 0$. The integrations of these constraints restrict the function $g(t, r)$ to be independent of the variable t , i.e. $g(t, r) = f(r)$. Subsequently, the combination $(\mathcal{G}^t_t - \mathcal{G}^r_r) - \kappa_n^2(T^t_t - T^r_r) = 0$ gives rise to two different possibilities, namely $\delta(t, r) = \bar{\delta}(t)$ or

$$f(r) = k + \frac{r^2}{2(n-3)(n-4)\alpha}. \quad (3.7)$$

Let us first consider the latter case. Putting Eq. (3.7) in the left-hand side of the field equation (2.6), we obtain

$$\mathcal{G}^a_b = \left[\Lambda + \frac{(n-1)(n-2)}{8\alpha(n-3)(n-4)} - \frac{(n-2)\alpha\Theta}{2r^4} \right] \delta^a_b, \quad (3.8)$$

$$\mathcal{G}^i_j = \left[\Lambda + \frac{(n-1)(n-2)}{8\alpha(n-3)(n-4)} - \frac{(n-6)\alpha\Theta}{2r^4} \right] \delta^i_j. \quad (3.9)$$

Hence, for $C \neq 0$, we obtain the following constraints on the constants of the problem:

$$(n-2)c_1 + c_2 = 0, \quad C^2 = \frac{2g^2\alpha\Theta}{\kappa_n^2}, \quad 1 + \frac{8(n-3)(n-4)\alpha\Lambda}{(n-1)(n-2)} = 0 \quad (3.10)$$

with an arbitrary metric function $\delta(t, r)$. It is interesting to note that for $C = 0$ and $\Theta = 0$, this solution reduces to the vacuum solution obtained in [25].

On the other hand, in the case of $\delta(t, r) = \bar{\delta}(t)$, we can set $\bar{\delta}(t) \equiv 0$ without loss of generality by redefining the time coordinate. Then, the metric (2.12) reduces to the following simple spacetime with only one unknown function $f(r)$:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\gamma_{ij}dz^i dz^j. \quad (3.11)$$

The metric function $f(r)$ is obtained by integrating the gravitational equations $\mathcal{G}^a_b = \kappa_n^2 T^a_b$ as

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}} \left(1 \mp \sqrt{1 + 4\tilde{\alpha}\tilde{\Lambda} + \frac{4\tilde{\alpha}\tilde{M}}{r^{n-1}} + \frac{2\tilde{\alpha}(\kappa_n^2 C^2 - 2g^2\tilde{\alpha}\tilde{\Theta})}{(n-5)g^2 r^4} - \frac{8\kappa_n^2\tilde{\alpha}C^4\{(n-2)c_1 + c_2\}}{(n-9)r^8}} \right), \quad (3.12)$$

where $\tilde{M} := 4\kappa_n^2 M / [(n-2)V_{n-2}^k]$, $\tilde{\alpha} := (n-3)(n-4)\alpha$, $\tilde{\Lambda} := 2\Lambda / [(n-1)(n-2)]$, $\tilde{\Theta} := \Theta / [(n-3)(n-4)]$, and M is a constant. In the asymptotically flat vacuum case ($k = 1$, $\Theta = 0$, $C = 0$, and $\Lambda = 0$ with the upper sign in (3.12)), M gives the Arnowitt-Deser-Misner (ADM) mass. We emphasize that Eq. (3.10) may be satisfied in this solution. Under the assumptions that $(D_a r)(D^a r) \neq 0$ and the relation

$$1 + \frac{8(n-3)(n-4)\alpha\Lambda}{(n-1)(n-2)} \neq 0, \quad (3.13)$$

the spacetime (3.11) with the metric function (3.12) is the unique form of the solution.

Various comments can be made concerning this solution. Firstly, there are two branches of solutions corresponding to the sign in front of the square root in Eq. (3.12), stemming from the quadratic nature of the field equations. Only the solution with the upper sign, that we call the GR branch, has a general relativistic (GR) limit as $\alpha \rightarrow 0$ given by

$$f(r) = k - \tilde{\Lambda}r^2 - \frac{\tilde{M}}{r^{n-3}} - \frac{\kappa_n^2 C^2}{2(n-5)g^2 r^2} + \frac{2\kappa_n^2 C^4\{(n-2)c_1 + c_2\}}{(n-9)r^6}. \quad (3.14)$$

(In contrast, there is only one branch of real solutions in the third-order Lovelock gravity [32].) Secondly, the metric function (3.12) reduces to the solutions obtained by Dotti and Gleiser [19] for $C = 0$, by Boulware and Deser, and independently by Wheeler [5] for $\Theta = 0$, $C = 0$, $k = 1$, and $\Lambda = 0$ and by Lorenz-Petzold and independently by Cai for $\Theta = 0$ and $C = 0$ [10, 12]. Lastly, we see that the metric function (3.12) is not well-defined for $C \neq 0$ with $n = 5$ or $n = 9$. However, it is shown in the next subsection that there is no magnetic solution in odd dimensions, namely we have $C \equiv 0$ for odd n .

3.2 Non-existence in odd dimensions

We have shown the possible form of the metric for the magnetic solution (3.12). This does not ensure that there exists a non-trivial magnetic components of the Faraday tensor satisfying Eqs. (2.11) and (2.19). Under the assumptions presented in the previous section, we prove the non-existence of magnetic solutions in any odd dimensions. The proof of this statement is given in [33] for a more general class of spacetimes but in order for the paper to be self-contained we present it here in a compact form. First we obtain $\det(F_{ij}) \equiv 0$ in odd dimensions by the anti-symmetric nature of F_{ij} , explicitly shown by $\det(F_{ij}) = \det(F_{ji}) = \det(-F_{ij}) = (-1)^{n-2} \det(F_{ij})$. Taking the determinant of Eq. (2.19), we obtain

$$\det(F_{il})\det(F_{jk})\det(\gamma^{lk}) = C^{2(n-2)}\det(\gamma_{ij}), \quad (3.15)$$

which gives $C \equiv 0$ for odd n . Combining the trace of Eq. (2.19), which is $F_{ij}F^{ij} = (n-2)C^2$, together with the fact that γ_{ij} is an Euclidean metric, we conclude $F_{ij} \equiv 0$ in any odd dimensions.

3.3 Exact magnetic solutions in even dimensions

Next, we show that there exists a non-trivial magnetic solution in even dimensions. We first review the monopole-type magnetic solution in four dimensions. In this case, the Faraday tensor $F_{\mu\nu}dx^\mu \wedge dx^\nu = Q_m h(\theta)d\theta \wedge d\phi$ is the solution for any k , where we adopt the coordinates on (K^2, γ_{ij}) such that $\gamma_{ij}dz^i dz^j = d\theta^2 + h(\theta)^2 d\phi^2$, where

$$h(\theta) := \begin{cases} \sin(\theta) & \text{for } k = 1, \\ 1 & \text{for } k = 0, \\ \sinh(\theta) & \text{for } k = -1. \end{cases} \quad (3.16)$$

Note that in this case, the constant C is given by $C^2 = Q_m^2$.

In higher dimensions, the existence problem of the magnetic solution is highly non-trivial even in general relativity without the gauge-correction terms except for $k = 1$ with $\Theta = 0$ where the magnetic solutions are ruled out. Indeed, for $k = 1$ with $\Theta = 0$, the manifold (K^{n-2}, γ_{ij}) is maximally symmetric with positive curvature. In the standard Maxwell case, F_{ij} is a harmonic 2-form on (K^{n-2}, γ_{ij}) since it satisfies $D_j F^{ij} = 0$, where D_i is the covariant derivative on (K^{n-2}, γ_{ij}) . It is well-known that if the manifold (K^{n-2}, γ_{ij}) is compact and its second Betti number is zero, $F_{ij} \equiv 0$ is satisfied. This is sufficient to prove the non-existence of magnetic solutions with $k = 1$ and $\Theta = 0$ for $n \geq 5$ [34]. In the presence of the gauge-correction terms, this argument is no longer valid since the tensor F_{ij} is not necessarily a harmonic 2-form.

In what follows, we establish the existence of magnetic solutions with or without the gauge-correction terms for some special class of Einstein space by extending the standard four-dimensional monopole result in higher even dimensions. We consider the Einstein space given as the $(n-2)$ -dimensional product space of $(n-2)/2$ two-dimensional spaces of constant curvature $K^{n-2} \approx$

$\underbrace{K^2 \times \dots \times K^2}_{(n-2)/2}$ with the same constant warp factor. The metric on such (K^{n-2}, γ_{ij}) is given by

$$\gamma_{ij} = \frac{1}{n-3} \text{diag}(\bar{\gamma}_{a_1 b_1}, \bar{\gamma}_{a_2 b_2}, \dots, \bar{\gamma}_{a_{(n-2)/2} b_{(n-2)/2}}), \quad (3.17)$$

$$\bar{\gamma}_{a_\sigma b_\sigma} dz^{a_\sigma} dz^{b_\sigma} = d\theta_\sigma^2 + h(\theta_\sigma)^2 d\phi_\sigma^2. \quad (3.18)$$

where $\sigma = 1, 2, \dots, (n-2)/2$, for which we have $k = \pm 1, 0$ and $\Theta = 2(n-3)(n-4)k^2$. As said before, the manifold (K^{n-2}, γ_{ij}) is maximally symmetric for $k = 0$. Since a two-dimensional space of non-positive constant curvature can be compactified by certain identifications, the $(n-2)$ -dimensional Einstein space $K^{n-2} \approx K^2 \times \dots \times K^2$ can be also compactified.

On the above Einstein space, it is easy to show that the following Faraday tensor satisfies the condition (2.19):

$$F_{\mu\nu} dx^\mu \wedge dx^\nu = Q_m \sum_{\sigma=1}^{(n-2)/2} (h(\theta_\sigma) d\theta_\sigma \wedge d\phi_\sigma), \quad (3.19)$$

where $Q_m^2 \equiv C^2/(n-3)^2$. This Faraday tensor satisfies the gauge-corrected Maxwell equation (2.11) as shown below.

The gauge-corrected Maxwell equation (2.11) can be written as

$$0 = \partial_\nu \left[\sqrt{-\det(g_{\mu\nu})} \left(-\frac{1}{g^2} F^{\mu\nu} + 8c_1 \mathcal{F} F^{\mu\nu} + 8c_2 F^{\mu\rho} F_{\rho\sigma} F^{\sigma\nu} \right) \right]. \quad (3.20)$$

For our metric, we obtain

$$-\det(g_{\mu\nu}) = \frac{r^{2(n-2)}}{(n-3)^{n-2}} \prod_{\sigma=1}^{(n-2)/2} h(\theta_\sigma)^2. \quad (3.21)$$

Hence, for $\mu = a$, the gauge-corrected Maxwell equation gives

$$0 = \partial_b \left[r^{n-2} \left(-\frac{1}{g^2} F^{ab} + 8c_1 \mathcal{F} F^{ab} + 8c_2 F^{ad} F_{df} F^{fb} \right) \right], \quad (3.22)$$

which is trivially satisfied for the magnetic case. For $\mu = a_\sigma$, we obtain

$$0 = \partial_{b_\sigma} \left[h(\theta_\sigma) \left(-\frac{1}{g^2} F^{a_\sigma b_\sigma} + 8c_1 \mathcal{F} F^{a_\sigma b_\sigma} + 8c_2 F^{a_\sigma d_\sigma} F_{d_\sigma f_\sigma} F^{f_\sigma b_\sigma} \right) \right]. \quad (3.23)$$

Using the following expressions:

$$F_{\theta_\sigma \phi_\sigma} = Q_m h, \quad F^{\theta_\sigma \phi_\sigma} = \frac{(n-3)^2}{r^4 h} Q_m, \quad (3.24)$$

$$\mathcal{F} = \frac{(n-2)(n-3)^2}{r^4} Q_m^2, \quad (3.25)$$

we show that inside the bracket in Eq. (3.23) is independent from z^{b_σ} . Hence, Eq. (3.23) is also satisfied.

We finally close this section by briefly commenting about the existence of the dyonic solution, that is the solution with both electric and magnetic charges. Although it is difficult to obtain the explicit form of the metric function in the dyonic case with the gauge corrections, this task is render possible in the absence of these terms. The solution in this case is given by

$$f(r) = k + \frac{r^2}{2\tilde{\alpha}} \left(1 \mp \sqrt{1 + 4\tilde{\alpha}\tilde{\Lambda} + \frac{4\tilde{\alpha}\tilde{M}}{r^{n-1}} - \frac{4\kappa_n^2\tilde{\alpha}Q_e^2}{(n-2)(n-3)g^2r^{2(n-2)}} + \frac{2\tilde{\alpha}(\kappa_n^2C^2 - 2g^2\tilde{\alpha}\tilde{\Theta})}{(n-5)g^2r^4}} \right), \quad (3.26)$$

and the non-zero electric component of the Faraday tensor is

$$F_{tr} = \frac{Q_e}{r^{n-2}}, \quad (3.27)$$

where Q_e is a constant corresponding to the electric charge.

4. Properties of the magnetic black holes with gauge corrections

In this section, we analyze the properties of the magnetic solution (3.11) with Eq. (3.12). We first point out that the coupling constants of the gauge-correction terms c_1 and c_2 appear in the metric function only through the combination $(n-2)c_1 + c_2$. This shows a sharp contrast with the electric case, in which they appear in the following more rigid form $2c_1 + c_2$ [14]. It is also appealing to note that in the purely magnetic case, the power of the potential of the Maxwell term as well as the gauge-correction term appearing in the metric is independent of the number of dimensions. This is clearly in contrast with the solution in the purely electric case. We note that, in the monopole type solution with the Yang-Mills field, the power of the matter term in the metric function is also constant for $n \geq 6$ [35].

4.1 Curvature singularities

In the spacetime given by (3.12), there are at most two classes of curvature singularities. There is a curvature singularity localized at the center $r = 0$ while the other is at $r = r_b$, where r_b corresponds to the possible zero of the square-root piece of the metric function (3.12). Both at $r = 0$ and $r = r_b$, the Kretschmann invariant

$$\begin{aligned} K &:= R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \\ &= \left(\frac{d^2f}{dr^2}\right)^2 + \frac{2(n-2)}{r^2}\left(\frac{df}{dr}\right)^2 + \frac{2(n-2)(n-3)}{r^4}(k-f)^2 \end{aligned} \quad (4.1)$$

blows up. The latter is called the branch singularity since two branches of solutions meet there. The branch singularity is a characteristic singularity in higher-curvature gravity located at a finite physical radius in general. As a direct consequence of the existence of branch singularity is that the domain of the radial coordinate r can not be extended from 0 to ∞ . The appearance of the branch singularity sharply depends on the parameters of the solution, and the location $r = r_b$ is given by solving the

following algebraic equation $B(r_b) = 0$, where

$$B(r_b) := 1 + 4\tilde{\alpha}\tilde{\Lambda} + \frac{4\tilde{\alpha}\tilde{M}}{r_b^{n-1}} + \frac{2\tilde{\alpha}(\kappa_n^2 C^2 - 2g^2\tilde{\alpha}\tilde{\Theta})}{(n-5)g^2 r_b^4} - \frac{8\kappa_n^2\tilde{\alpha}C^4\{(n-2)c_1 + c_2\}}{(n-9)r_b^8}. \quad (4.2)$$

The physical domain of the radial coordinate r is given by $B(r) > 0$.

Here we only consider the physically reasonable situations in which $\alpha > 0$, $1 + 4\tilde{\alpha}\tilde{\Lambda} \geq 0$, and $M \geq 0$ are satisfied. The first condition is imposed by string theory, while the second inequality ensures the existence of the maximally symmetric solution. The last condition means that the parameter M which is assimilated to the mass is positive. With this respect, it is a non-trivial issue to see whether the parameter M can be identified as the mass of a black hole in the present case because the spacetime has a non-trivial boundary. However, in the vacuum case, M coincides with the well-defined quasi-local mass and satisfies the first law of the black-hole thermodynamics together with the Wald entropy [20]. For these reasons, we call M the mass parameter. Finally, it is simple to see that under the condition $(n-9)[(n-2)c_1 + c_2] \leq 0$ with a sufficiently large value of the magnetic constant C^2 , the solution is free from branch singularities.

4.2 Asymptotic structure

Let us next consider the asymptotic structure of our solution, that is the behavior for $r \rightarrow \infty$. For $\Theta = 0$, this spacetime is at least locally asymptotically flat or (anti-)de Sitter ((A)dS) for $\lambda = 0$ or $\lambda(<) > 0$, respectively, in the sense that

$$R^{\mu\nu}_{\sigma\rho}|_{r \rightarrow \infty} = \lambda (\delta^\mu_\sigma \delta^\nu_\rho - \delta^\mu_\rho \delta^\nu_\sigma), \quad (4.3)$$

$$\lambda := -\frac{1}{2\tilde{\alpha}} \left(1 \mp \sqrt{1 + 4\tilde{\alpha}\tilde{\Lambda}} \right). \quad (4.4)$$

In four dimensions, in which $\Theta = 0$, the magnetic term respects the fall-off conditions to the asymptotically flat or AdS regions. Then, for $k = 1$, M corresponds to the Arnowitt-Deser-Misner (ADM) mass and to the Abbott-Deser (AD) mass in the asymptotically flat and AdS cases, respectively. In higher dimensions, on the other hand, the fall-off rate of the magnetic and the Weyl terms in the metric function is slower than the mass term.

The contribution of the higher-order gauge corrections decays more rapidly for $r \rightarrow \infty$ than the Maxwell term. On the other hand, the gauge-correction term dominates around the center $r \rightarrow 0$ in the generic case and its contribution is quite sensitive to the sign of $(n-2)c_1 + c_2$. In the general relativistic case (3.14), unlike in four dimensions, the magnetic term contributes as the attractive force in higher dimensions while the higher-order gauge corrections give the repulsive (attractive) force depending on the sign of the constant $[(n-2)c_1 + c_2]/(n-9)$. As a result, the global structure of the spacetime can be quite different from the standard Reissner-Nordström case. Finally, we close this section by stressing that through a fine-tuning between the parameters such as $(n-2)c_1 + c_2 = 0$, the gauge corrections do not appear in the metric. In addition, if the magnetic constant has a very precise value $C^2 = 2g^2\tilde{\alpha}\tilde{\Theta}/\kappa_n^2$, the metric function (3.12) is the same as the generalized Boulware-Deser-Wheeler solution.

4.3 Energy conditions

In fact, the sign of $(n-2)c_1 + c_2$ is closely related to the energy condition. The energy-momentum tensor of our matter field has the diagonal form as $T^\mu_\nu = \text{diag}(-\mu, p_r, p_t, p_t, \dots)$. The physical interpretations of μ , p_r and p_t are the energy density, radial pressure and tangential pressure, respectively. The weak energy condition (WEC) implies $\mu \geq 0$, $p_r + \mu \geq 0$, and $p_t + \mu \geq 0$, while the dominant energy condition (DEC) implies $\mu \geq 0$, $-\mu \leq p_r \leq \mu$, and $-\mu \leq p_t \leq \mu$. The null energy condition (NEC) implies $p_r + \mu \geq 0$, and $p_t + \mu \geq 0$ [36, 37]. Note that DEC implies WEC and WEC implies NEC.

For our matter field, the corresponding energy density, radial pressure, and the tangential pressure are

$$\begin{aligned}\mu &= \frac{(n-2)C^2}{4g^2r^4} - \frac{(n-2)C^4\{(n-2)c_1 + c_2\}}{r^8}, \\ p_r &= -\frac{(n-2)C^2}{4g^2r^4} + \frac{(n-2)C^4\{(n-2)c_1 + c_2\}}{r^8}, \\ p_t &= -\frac{(n-6)C^2}{4g^2r^4} + \frac{(n-10)C^4\{(n-2)c_1 + c_2\}}{r^8},\end{aligned}$$

from which we obtain

$$\begin{aligned}\mu + p_r &= 0, \\ \mu - p_r &= \frac{(n-2)C^2}{2g^2r^4} - \frac{2(n-2)C^4\{(n-2)c_1 + c_2\}}{r^8}, \\ \mu + p_t &= \frac{C^2}{g^2r^4} - \frac{8C^4\{(n-2)c_1 + c_2\}}{r^8}, \\ \mu - p_t &= \frac{(n-4)C^2}{2g^2r^4} - \frac{2(n-6)C^4\{(n-2)c_1 + c_2\}}{r^8}.\end{aligned}$$

Hence, it is clear that if $(n-2)c_1 + c_2 \leq 0$, the DEC is satisfied for any positive r while if $(n-2)c_1 + c_2 > 0$, the NEC is violated near $r = 0$.

4.4 Black hole configurations

Now we clarify the parameter region where the solution represents a black hole. A Killing horizon is given by $r = r_h$ such that $f(r_h) = 0$. An outer Killing horizon is defined by $f(r_h) = 0$ with $df/dr(r_h) > 0$. On the other hand, an inner and degenerate Killing horizons are characterized by $df/dr(r_h) < 0$ and $df/dr(r_h) = 0$, respectively. A black hole is defined by an event horizon, which is an outermost outer Killing horizon if there exists null infinity. Notice that an outermost degenerate Killing horizon with $d^2f/dr^2(r_h) > 0$ may also be an event horizon.

For this purpose, the \tilde{M} - r_h diagram is quite useful. (See [38] for the analysis with or without the Maxwell electric charge in the case of the maximally symmetric horizon.) The \tilde{M} - r_h relation is

obtained from the equation $f(r_h) = 0$ as

$$\begin{aligned}\tilde{M} &= -\tilde{\Lambda}r_h^{n-1} + kr_h^{n-3} - \frac{[\kappa_n^2 C^2 - 2g^2 \tilde{\alpha}\{(n-5)k^2 + \tilde{\Theta}\}]}{2(n-5)g^2} r_h^{n-5} + \frac{2\kappa_n^2 C^4 \{(n-2)c_1 + c_2\}}{n-9} r_h^{n-9}, \\ &=: \tilde{M}_h(r_h).\end{aligned}\quad (4.5)$$

On the other hand, the \tilde{M} - r_b relation is obtained from $B(r_b) = 0$ as

$$\begin{aligned}\tilde{M} &= -\frac{1}{4\tilde{\alpha}}(1 + 4\tilde{\alpha}\tilde{\Lambda})r_b^{n-1} - \frac{(\kappa_n^2 C^2 - 2g^2 \tilde{\alpha}\tilde{\Theta})}{2(n-5)g^2} r_b^{n-5} + \frac{2\kappa_n^2 C^4 \{(n-2)c_1 + c_2\}}{(n-9)} r_b^{n-9}, \\ &=: \tilde{M}_b(r_b).\end{aligned}\quad (4.6)$$

We calculate

$$\tilde{M}_h(r) - \tilde{M}_b(r) = \frac{r^{n-5}(r^2 + 2\tilde{\alpha}k)^2}{4\tilde{\alpha}}, \quad (4.7)$$

and hence $\tilde{M}_h \geq \tilde{M}_b$ is satisfied for $\alpha > 0$ with equality holding at $r = 0$ as well as for $r^2 = -2\tilde{\alpha}k$ as long as $\tilde{\alpha}k < 0$.

The number of horizons and the existence of the branch singularity for the given mass M are totally understood by the functional forms of $\tilde{M}_h(r)$ and $\tilde{M}_b(r)$, respectively. However, the shape of the two curves $\tilde{M} = \tilde{M}_h(r)$ and $\tilde{M} = \tilde{M}_b(r)$ depends on the parameters in a complicated manner and it is almost hopeless to provide a complete classification. However, since this work is motivated by the low-energy action of string theory (2.4), we focus our attention on the case with $\alpha \geq 0$, $\Lambda = 0$, $(n-2)c_1 + c_2 \leq 0$ and with $k = 1$ (and hence $\Theta = 2(n-3)(n-4)$). In this case, the previous expressions reduce to

$$\tilde{M}_h(r) = r^{n-3} - \frac{q^2 - 2(n-3)\tilde{\alpha}}{2(n-5)} r^{n-5} - \frac{2q^4 d^2}{n-9} r^{n-9}, \quad (4.8)$$

$$\tilde{M}_b(r) = -\frac{1}{4\tilde{\alpha}} r^{n-1} - \frac{q^2 - 4\tilde{\alpha}}{2(n-5)} r^{n-5} - \frac{2q^4 d^2}{n-9} r^{n-9}, \quad (4.9)$$

$$q^2 := \frac{\kappa_n^2 C^2}{g^2}, \quad d^2 := -\frac{g^4 \{(n-2)c_1 + c_2\}}{\kappa_n^2}. \quad (4.10)$$

For later purpose, let us compute the first and second derivatives:

$$\frac{d\tilde{M}_h}{dr} = (n-3)r^{n-4} - \frac{q^2 - 2(n-3)\tilde{\alpha}}{2} r^{n-6} - 2q^4 d^2 r^{n-10}, \quad (4.11)$$

$$\frac{d^2 \tilde{M}_h}{dr^2} = (n-3)(n-4)r^{n-5} - \frac{(n-6)[q^2 - 2(n-3)\tilde{\alpha}]}{2} r^{n-7} - 2(n-10)q^4 d^2 r^{n-11}, \quad (4.12)$$

$$\frac{d\tilde{M}_b}{dr} = -\frac{n-1}{4\tilde{\alpha}} r^{n-2} - \frac{q^2 - 4\tilde{\alpha}}{2} r^{n-6} - 2q^4 d^2 r^{n-10}, \quad (4.13)$$

$$\frac{d^2 \tilde{M}_b}{dr^2} = -\frac{(n-1)(n-2)}{4\tilde{\alpha}} r^{n-3} - \frac{(n-6)[q^2 - 4\tilde{\alpha}]}{2} r^{n-7} - 2(n-10)q^4 d^2 r^{n-11}. \quad (4.14)$$

We will see that the existence or absence of the horizon depends on the parameters. The solution with horizons belongs the GR branch because $f(r) > 0$ is satisfied and there is no horizon in the

non-GR branch for $\alpha > 0$ with $k = 1$. For $\alpha > 0$ with $k = 1$, the branch singularity is in the untrapped region defined by $f(r) > 0$. The asymptotic region $r \rightarrow \infty$ is in the untrapped region since $\lim_{r \rightarrow \infty} f(r) = 1$ is satisfied for $k = 1$ and $\Lambda = 0$ in the GR branch.

In order to clarify the effects of the higher-order correction terms, we will study four cases separately in the following subsections. There we adopt the unit such that $\tilde{\alpha} = 1$ for $\alpha > 0$. For $(n - 2)c_1 + c_2 \neq 0$, we adopt the unit in addition such that $d^2 = 1$.

4.4.1 General relativity without gauge corrections

First we consider the simplest case, namely the general relativistic case without gauge corrections. The \tilde{M} - r_h diagram given by Eq. (4.8) with $\alpha = d = 0$ is qualitatively different between $n = 4$ and $n \geq 6$. Also, it is different between $q^2 = 0$ and $q^2 \neq 0$ for each n . (See Fig. 1.)

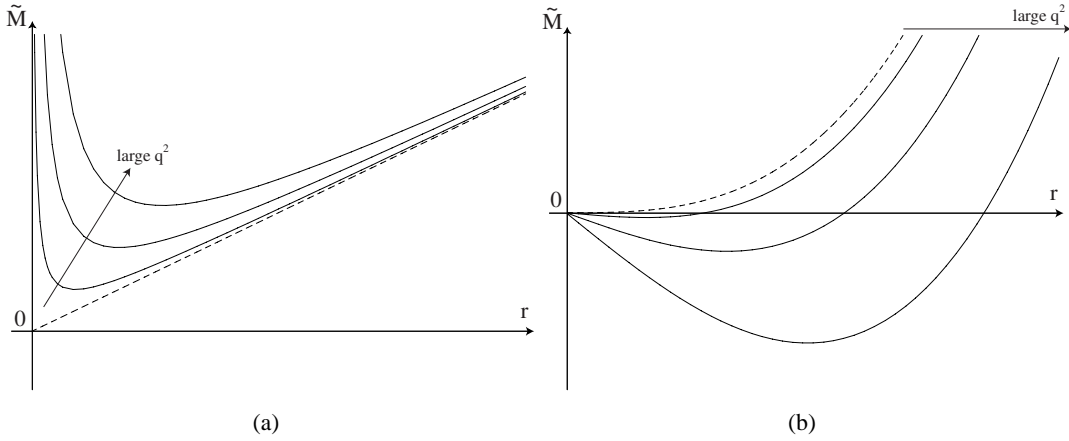


Figure 1: The function $\tilde{M} = \tilde{M}_h(r)$ in the positive-curvature case without a cosmological constant and gauge corrections in general relativity ($k = 1$, $\Lambda = 0$, $\alpha = 0$, and $(n - 2)c_1 + c_2 = 0$). The parameter dependence on q^2 is shown for (a) $n = 4$ and (b) $n = 6$. A dashed curve corresponds to the case with $q^2 = 0$. The graph for $n \geq 8$ is qualitatively the same as $n = 6$.

For $n = 4$, the situation is the same as the Schwarzschild or the Reissner-Nordström solution. For $n \geq 6$ with $q^2 = 0$, there is one outer horizon for $\tilde{M} > 0$, while there is no horizon for $\tilde{M} \leq 0$. For $n \geq 6$ with $q^2 > 0$, $\tilde{M} = \tilde{M}_h(r)$ has one local minimum at $\tilde{M} = \tilde{M}_{\text{ex}} (< 0)$. There is one outer horizon for $\tilde{M} \geq 0$, one outer and one inner horizons for $0 > \tilde{M} > \tilde{M}_{\text{ex}}$, one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}}$.

4.4.2 General relativity with gauge corrections

Next we consider the effect of the gauge-correction terms in general relativity. The \tilde{M} - r_h diagrams given by Eq. (4.8) with $\alpha = 0$ and $d^2 = 1$ are shown in Fig. 2. For $n = 4$, the situation is qualitatively the same as the case without gauge corrections. For $n \geq 6$ with $q^2 = 0$, there is one outer horizon for $\tilde{M} > 0$, while there is no horizon for $\tilde{M} \leq 0$. For $n = 6, 8$ with $q^2 > 0$, $\tilde{M} = \tilde{M}_h(r)$ has one local minimum at $\tilde{M} = \tilde{M}_{\text{ex}} (> 0)$. There are one outer and one inner horizons for $\tilde{M} > \tilde{M}_{\text{ex}}$,

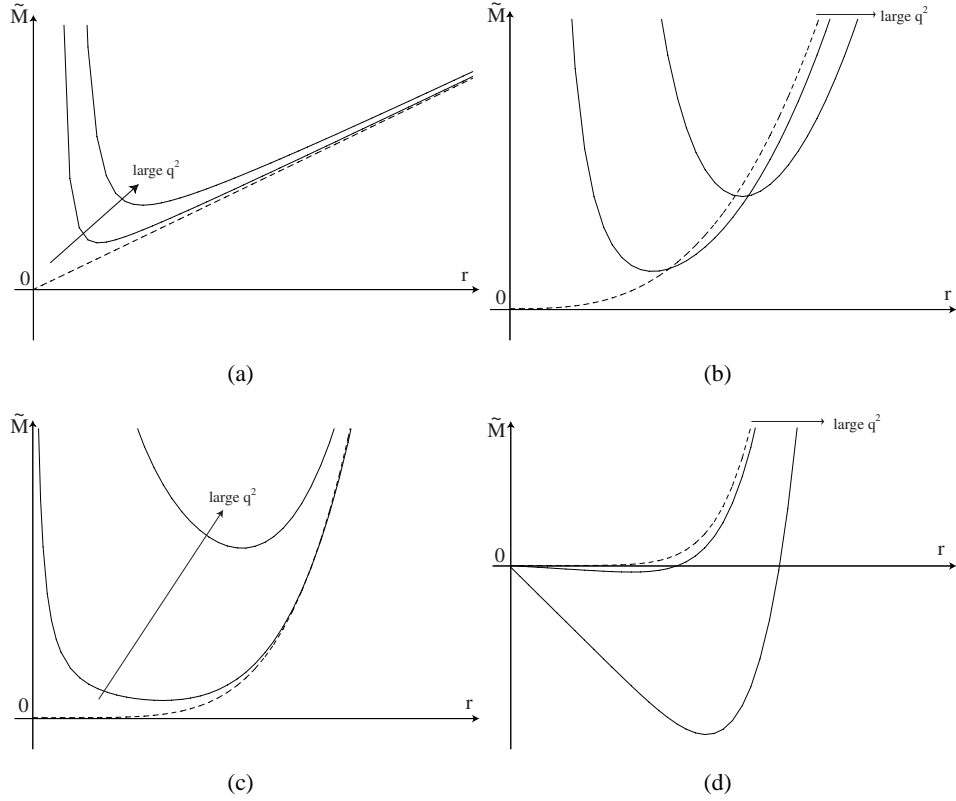


Figure 2: The function $\tilde{M} = \tilde{M}_h(r)$ in the positive-curvature case without a cosmological constant in general relativity but with gauge corrections ($k = 1$, $\Lambda = 0$, $\alpha = 0$, and $(n-2)c_1 + c_2 < 0$). The parameter dependence on q^2 is shown for (a) $n = 4$, (b) $n = 6$, (c) $n = 8$, and (d) $n = 10$. A dashed curve corresponds to the case with $q^2 = 0$. The graph for $n \geq 12$ is qualitatively the same as $n = 10$.

one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}}$. For $n \geq 10$ with $q^2 > 0$, $\tilde{M} = \tilde{M}_h(r)$ has one local minimum at $\tilde{M} = \tilde{M}_{\text{ex}} (< 0)$. There is one outer horizon for $\tilde{M} \geq 0$, one outer and one inner horizons for $0 > \tilde{M} > \tilde{M}_{\text{ex}}$, one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}}$.

4.4.3 Einstein-Gauss-Bonnet gravity without gauge corrections

Now we consider the effect of the Gauss-Bonnet term for $n \geq 6$. The most drastic change is the existence of the branch singularity. In the presence of the branch singularity for given M , we only consider the domain of r connecting to the asymptotic region, namely $r_b < r < \infty$.

We first consider the case without gauge corrections. The \tilde{M} - r_h and \tilde{M} - r_b diagrams, given respectively by Eqs. (4.8) and (4.9) with $\tilde{\alpha} = 1$ and $d^2 = 0$, are shown in Fig. 3. The graphs are qualitatively the same for any $n \geq 6$. For $0 \leq q^2 \leq 2(n-3)$, $\tilde{M} = \tilde{M}_h(r)$ is monotonically increasing from $\tilde{M} = 0$. For $q^2 > 2(n-3)$, $\tilde{M} = \tilde{M}_h(r)$ has one local minimum at $\tilde{M} = \tilde{M}_{\text{ex(h)}} (< 0)$. For

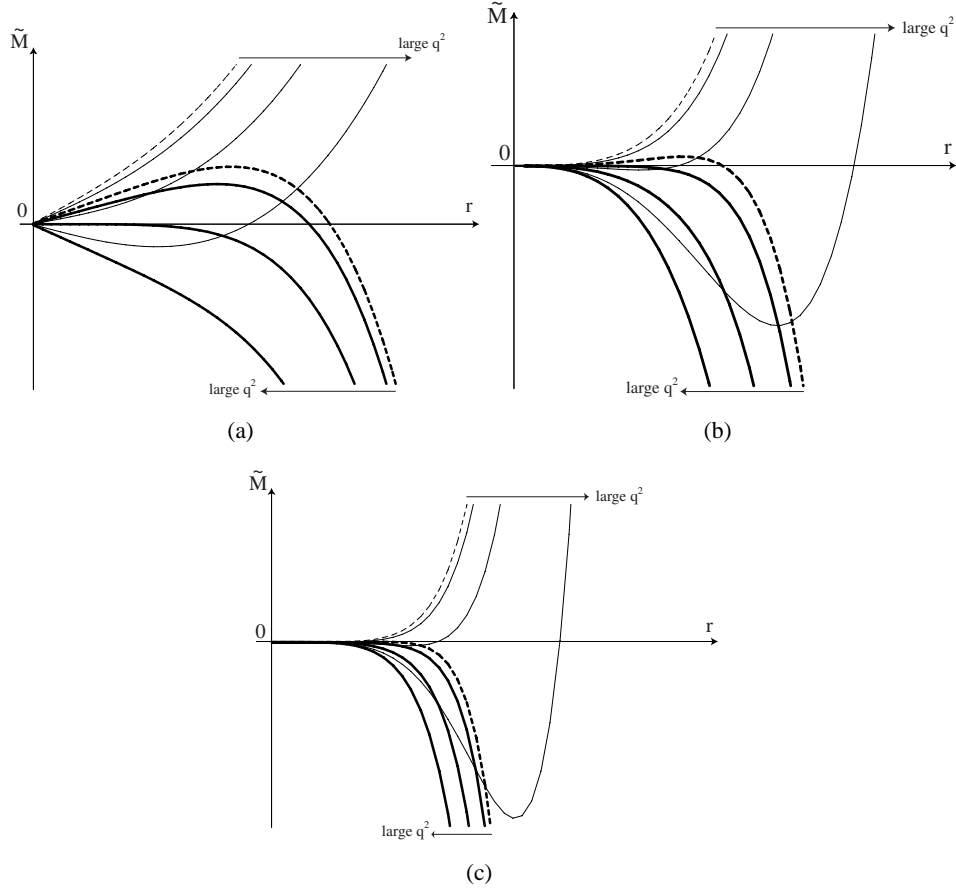


Figure 3: The functions $\tilde{M} = \tilde{M}_h(r)$ and $\tilde{M} = \tilde{M}_b(r)$ in the positive-curvature case without a cosmological constant and gauge corrections in Einstein-Gauss-Bonnet gravity ($k = 1$, $\alpha > 0$, $\Lambda = 0$, and $(n-2)c_1 + c_2 = 0$). The parameter dependence on q^2 is shown for (a) $n = 6$, (b) $n = 8$, and (c) $n = 10$. A thin and a thick curves correspond to $\tilde{M} = \tilde{M}_h(r)$ and $\tilde{M} = \tilde{M}_b(r)$, respectively. The dashed curves correspond to the case with $q^2 = 0$. The physical domain of r is $\tilde{M} > \tilde{M}_b$ and $\tilde{M} = \tilde{M}_b$ is in the untrapped region. The graph for $n \geq 12$ is qualitatively the same as $n = 10$.

$0 \leq q^2 < 4$, $\tilde{M} = \tilde{M}_b(r)$ has one local maximum at $\tilde{M} = \tilde{M}_{\text{ex}(b)}(> 0)$. For $q^2 \geq 4$, $\tilde{M} = \tilde{M}_b(r)$ is monotonically decreasing from $\tilde{M} = 0$.

Hence, for $0 \leq q^2 < 4$, there is one outer horizon for $\tilde{M} > \tilde{M}_{\text{ex}(b)}$ and no horizon for $\tilde{M} \leq \tilde{M}_{\text{ex}(b)}$. For $4 \leq q^2 \leq 2(n-3)$, there is one outer horizon for $\tilde{M} > 0$ and no horizon for $\tilde{M} \leq 0$. For $q^2 > 2(n-3)$, there is one outer horizon for $\tilde{M} \geq 0$, one outer and one inner horizons for $0 > \tilde{M} > \tilde{M}_{\text{ex}(h)}$, one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}(h)}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}(h)}$.

4.4.4 Einstein-Gauss-Bonnet gravity with gauge corrections

We finally consider the case where both the Gauss-Bonnet and gauge-correction terms are present. The \tilde{M} - r_h and \tilde{M} - r_b diagrams are given respectively by Eqs. (4.8) and (4.9) with $\tilde{\alpha} = 1$ and $d^2 = 1$

and the parameter dependence is rather complicated.

We first analyze the behavior of $\tilde{M} = \tilde{M}_b$ with the help of its derivative Eq. (4.13). It is simple to see that an extremum exists if

$$(q^2 - 4)^2 - 8(n - 1)q^4 > 0. \quad (4.15)$$

However, the left-hand side of the above inequality can not be positive for $n \geq 2$ and hence there is no extremum and $\tilde{M} = \tilde{M}_b$ is monotonic. Also, it is seen that $\lim_{r \rightarrow \infty} \tilde{M}_b(r) = -\infty$ and $\lim_{r \rightarrow \infty} \tilde{M}_h(r) = +\infty$. Near $r = 0$, we obtain $\lim_{r \rightarrow 0} \tilde{M}_b(r) = +\infty$, $\lim_{r \rightarrow 0} \tilde{M}_h(r) = +\infty$ for $n \leq 8$, while $\lim_{r \rightarrow 0} \tilde{M}_b(r) = 0$, $\lim_{r \rightarrow 0} \tilde{M}_h(r) = 0$ for $n \geq 10$.

The behavior of $\tilde{M} = \tilde{M}_h$ can be better analyzed by its derivatives (4.11) and (4.12). Since the algebraic equation $d\tilde{M}_h/dr = 0$ is essentially cubic for r^2 , it is difficult to provide a rigorous argument about the behavior of $\tilde{M} = \tilde{M}_h$. However, the numerical plots of $\tilde{M} = \tilde{M}_h$ indicate that there is only one local minimum at $\tilde{M} = \tilde{M}_{\text{ex}}$, where $\tilde{M}_{\text{ex}} > 0$ and $\tilde{M}_{\text{ex}} < 0$ are satisfied for $n = 6, 8$ and $n \geq 10$, respectively.

The \tilde{M} - r_h and \tilde{M} - r_b diagrams are shown in Fig. 4. In the case of $q^2 = 0$, the graphs are the same as the case without gauge corrections (Fig. 3 with $q^2 = 0$). For $n = 6, 8$, there are one outer and one inner horizons for $\tilde{M} > \tilde{M}_{\text{ex}}$, one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}}$. For any \tilde{M} , there exists a branch singularity. For $n = 10$, there is one outer horizon for $\tilde{M} \geq 0$, one outer and one inner horizons for $0 > \tilde{M} > \tilde{M}_{\text{ex}}$, one degenerate horizon for $\tilde{M} = \tilde{M}_{\text{ex}}$, and no horizon for $\tilde{M} < \tilde{M}_{\text{ex}}$. The branch singularity exists for $\tilde{M} < 0$.

5. Summary and discussions

In the present paper, we have considered the $n(\geq 4)$ -dimensional Einstein-Gauss-Bonnet equations in presence of a cosmological constant with a matter source given by the Maxwell action with the F^4 gauge-correction terms build up with the Faraday tensor. This action without a cosmological constant is realized in the low-energy limit of a class of string theories. We have assumed that the spacetime geometry is given by a warped product $\mathcal{M}^2 \times \mathcal{K}^{n-2}$, where \mathcal{K}^{n-2} is a $(n - 2)$ -dimensional Einstein space satisfying a specific condition (2.15) and the orbit of the warp factor on \mathcal{K}^{n-2} is non-null.

Under a few reasonable assumptions, we have established the generalized Jebsen-Birkhoff theorem for the magnetic solution which fixes the metric function in a unique form. Using a simple geometric argument, we have established the non-existence of such magnetic solutions in any odd dimensions. In even dimensions, we have obtained magnetic solutions in the case where \mathcal{K}^{n-2} is a product manifold of $(n - 2)/2$ two-dimensional maximally symmetric spaces with the same constant warp factors.

The coupling constants of the gauge-correction terms appear in the metric function in the form of $(n - 2)c_1 + c_2$ and the gauge-correction term converges to zero rapidly for $r \rightarrow \infty$, while it dominates in the short distance for $n \leq 8$. We have clarified whether the solution represents a black hole or not depending on the parameters in the case of $k = 1$, $\Lambda = 0$, $\alpha \geq 0$, $(n - 2)c_1 + c_2 \leq 0$, which is the most important case in direct relation with the string viewpoints. We have established that the existence of black hole configurations is not only tied to the existence of the gauge-correction terms,

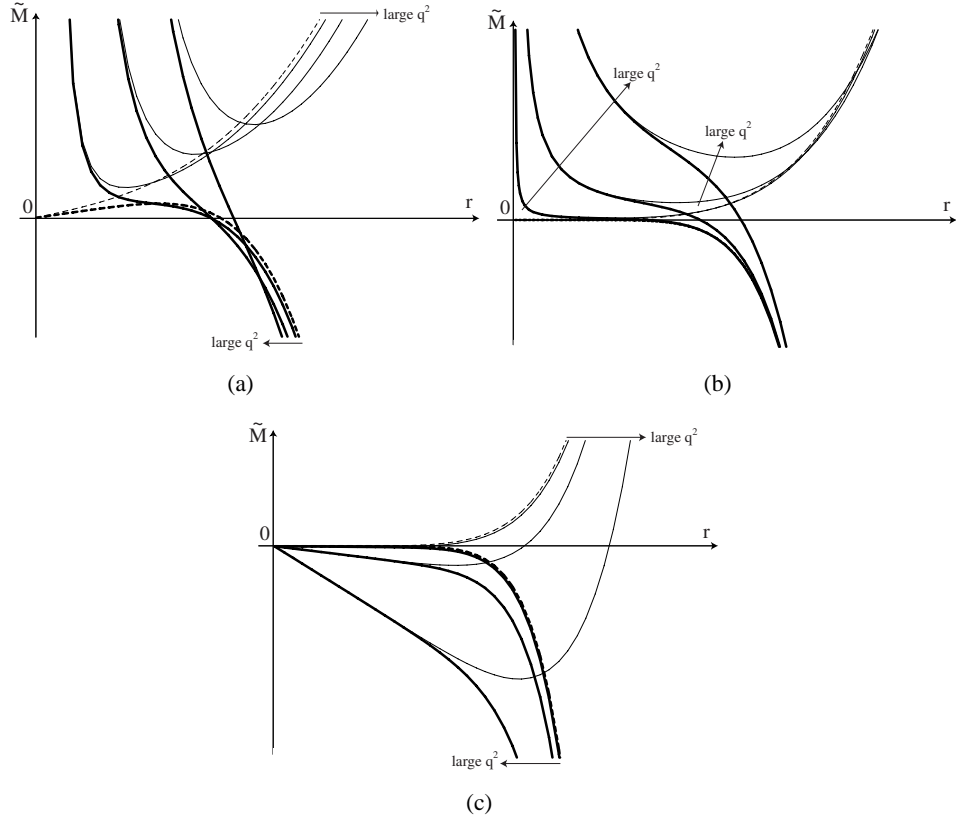


Figure 4: The functions $\tilde{M} = \tilde{M}_h(r)$ and $\tilde{M} = \tilde{M}_b(r)$ in the positive-curvature case without a cosmological constant in Einstein-Gauss-Bonnet gravity ($k = 1$, $\alpha > 0$, $\Lambda = 0$, and $(n - 2)c_1 + c_2 < 0$). The parameter dependence on q^2 is shown for (a) $n = 6$, (b) $n = 8$, and (c) $n = 10$. A thin and a thick curves correspond to $\tilde{M} = \tilde{M}_h(r)$ and $\tilde{M} = \tilde{M}_b(r)$, respectively. The dashed curves correspond to the case with $q^2 = 0$. The physical domain of r is $\tilde{M} > \tilde{M}_b$ and $\tilde{M} = \tilde{M}_b$ is in the untrapped region. The graph for $n \geq 12$ is qualitatively the same as $n = 10$.

but also to the number of spacetime dimensions. In the presence of the gauge-correction terms, the qualitative properties of the magnetic black hole is rather different if the even dimension $n \leq 8$ or if $n \geq 10$. This is not only because the power of the gauge-correction term in the metric function (3.12) becomes smaller than the mass term for $n \geq 10$, but also because the sign of the gauge-correction term is different for $n \leq 8$ and $n \geq 10$.

As a future task, the black-hole thermodynamics of our magnetic black hole is important. In Einstein-Gauss-Bonnet gravity, this subject have been intensively investigated with or without the Maxwell electric charge in the case where \mathcal{K}^{n-2} is maximally symmetric. The effect of the Weyl term on the thermodynamical stability has been recently analyzed for the Dotti-Gleiser vacuum black hole by one of the authors [20]. However, the thermodynamical aspect of magnetic black holes in higher dimensions has not been studied yet even in the standard Maxwell case in general relativity.

Another interesting problem would be to introduce a non-trivial dilaton since it naturally arises in

the low-energy limit of string theories. In this case, the existence of black-hole solutions will shed a new light on the semi-classical effects of string theory on black holes. These prospects presented here are left for possible future investigations. In the same spirit, we can also consider a general p -form coupled to a dilaton field as those that occur in standard supergravity theories. The advantage of these models is that the presence of the dilaton field permits to extend the notion of the electric-magnetic duality, and hence the existence of magnetic solutions is tied to the electric solutions.

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